RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 52, NUMBER 3 SEPTEMBER 1995

Controlling chaos by pinning neurons in a neural network

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(Received 11 April 1995)

Chaos control in an artificial neural network is achieved by "pinning" the state of a few neurons and adjusting the threshold value. The activity of the network, when the control is on, can be constant or periodic. The mechanism used is compared with other well-known algorithms of chaos control in low-dimensional maps. Some comments are made on possible similarities with memory retrieval in natural neural networks.

PACS number(s): 05.45.+b, 05.50.+q

Controlling chaos is presently an active line of research in the field of nonlinear dynamical systems. Ott, Grebogi, and Yorke [1] demonstrated chaos control using a feedback mechanism acting on one or more parameters of the chaotic system. Since then this technique has been applied to many low-dimensional maps and real physical systems [2–4]. The technique relies on a fundamental characteristic of chaotic systems, namely, their extreme sensitivity to initial conditions. By a suitable small perturbation, the chaotic system can be driven to any desired fixed or periodic state. Usually a chaotic attractor embeds a great number of periodic orbits. A carefully chosen perturbation, modifying a system parameter, can stabilize one of these periodic orbits. In a great majority of cases, chaos control has been applied to low-dimensional systems. However, controlling chaos in systems with a great number of degrees of freedom is important for practical uses. This has been performed with a coupled map lattice model [5] by fixing the values of some sites ("pinning"). In this case, there is a critical number of pinnings above which chaos can be controlled. Neural networks are spatiotemporal systems consisting of a great number of units (the neurons) that can be led to behave chaotically. It is desirable to know how to take a neural network from a chaotic to a fixed or periodic behavior by modulation of only a few degrees of freedom. This is the main concern of the present communication.

Artificial neural networks usually fall short as reliable models of high brain functions. The study of electroencephalograms (EEG's) of humans and animals [6,7] has revealed that the dynamics of brain activity is very rich and intricate, as would be expected. Techniques of analysis borrowed from the study of nonlinear dynamical systems have been of great utility in this line of research. A weak point in many artificial neural networks is their dynamical rigidity. This can be of minor concern when the network is used only for static memory retrieval but curtails the simulation of other interesting brain properties. Some mechanisms have been proposed to bypass these limitations, for instance, by adding controlled noise. In the present study we use local control or pinnings acting only on a few neurons. The intention here is to understand how these pinnings influence the dynamics of the network and how dense they have to be for controlling chaos.

In this report we use a typical model of neural network where the state of each neuron depends on the input from some other neurons that send signals modified by weights, the synapses. Although very simple, this model has an activity that can mimic the dynamics of real neural systems, as viewed through EEG time series. Results already published [8,9] show that the dynamics of this type of model can display a variety of behaviors. Acting on the threshold function, the activity of this network can be led to be fixed, periodic, intermittent, or chaotic. Now we are interested in taking this activity from chaotic to fixed or periodic by acting only on a few neurons while maintaining all other parameters fixed. We will show how this can be achieved and speculate on possible insights to be obtained on the mechanisms of memory and association of ideas in real neural networks.

Our model is a variation of the original model created by McCulloch and Pitts [10]. It consists of N neurons, each connected to n other randomly chosen neurons ($n \ll N$). A neuron is a simple binary unit, $x(t)$, that can take only two states: $x=0$ (inactive or quiescent) and $x=1$ (active or firing). The state of the neuron i at time step t is updated synchronously as

$$x_i(t)=\Theta\left(\sum_{j\in(i)}^{n} S_{ij}x_j(t-1)+T\right), \tag{1}$$

where (i) is the set of neurons sending input signals to neuron i and S_{ij} is the synaptic weight between neurons i and j. These weights can take any value from $+1$ and -1 and are randomly chosen in the present realization of the model. Positive weights correspond to excitatory synapses and negative weights to inhibitory synapses. They are not symmetric, that is, $S_{ij} \neq S_{ji}$. The threshold value T is taken as constant and is the same for all neurons. Θ is the Heaviside function: $\Theta(z)=1$ if $z \geq 0$ and $\Theta(z)=0$ if $z<0$. Our observable is the activity of the network, defined as

$$A(t)=\frac{\sum_{i=1}^{N} x_i(t)}{N}. \tag{2}$$

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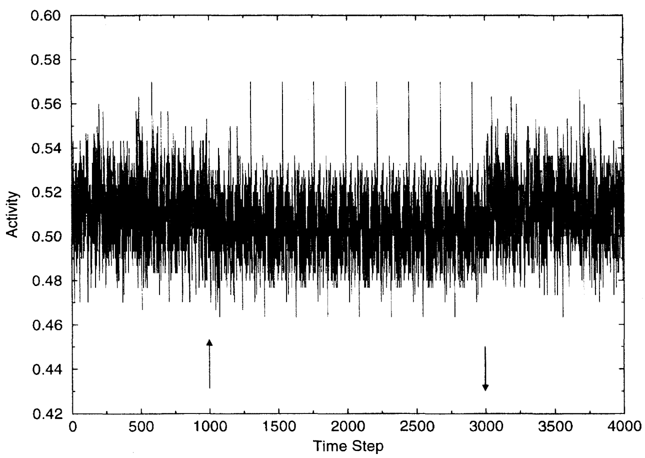

FIG. 1. Time series of the neural network activity with threshold $T=0$. Control is on from step 1000 to step 3000, with one neuron kept active. The controlled activity is periodic.

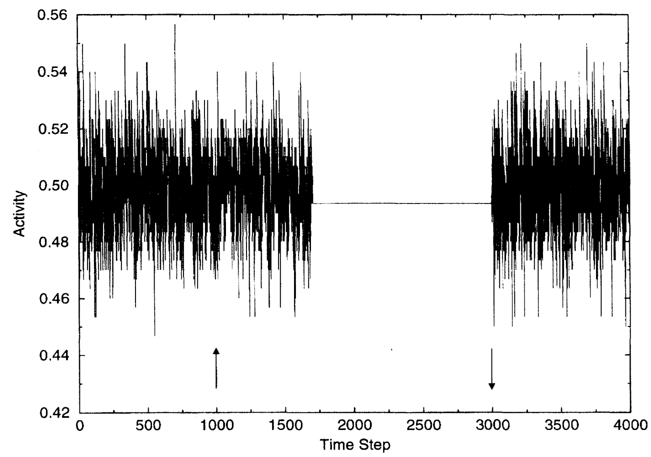

FIG. 2. Time series of the neural network activity with threshold $T=0$. Control is on from step 1000 to step 3000, with one neuron kept inactive. The controlled activity is constant.

The total number of neurons utilized in the simulations was $N=300$, with each neuron coupled to $n=10$ other randomly chosen neurons.

Depending on the value of the threshold T, the activity of the network can be constant, periodic, or chaotic. By chaotic we mean an activity time series that does not repeat itself for an extremely long time. A deterministic neural network with a finite number of neurons cannot display genuine chaos because the number of configurations is finite, although very large. As soon as any configuration is repeated, a cycle will take place. Here a chaotic behavior can be considered as a very long transient that eventually will settle as a periodic or fixed time series. However, for large values of N ($N=300$ is already large) this transient can last for an exceedingly long time and the dynamics can be considered chaotic. In order to control this chaotic behavior we act on one or a few neurons, either by fixing their states or feeding them with a periodic signal. In the present report we will consider only results obtained by fixing the state of a few neurons, that is, by pinning these neurons for some time. We are interested in studying how these pinnings influence the dynamics of the network. As will be shown, control can be achieved in certain cases with even only one controlling neuron with a fixed state. An analysis of the control process through the observation of the Poincaré map will then reveal that this process is an autonomous implementation of well-known control algorithms.

Figure 1 shows the effect of using only one neuron to take the entire network from chaotic to periodic behavior. In this example, the controlling neuron was chosen as tentative and its state was fixed as active while control was on. We will discuss later how this choice can be dependent on the synaptic weights. When control is turned on at time step 1000, the activity changes from chaotic to periodic, after a transient. Turning control off at time step 3000 restores the chaotic condition. Figure 2 shows another example of chaos control with only one neuron. After a transient, the activity goes from chaotic to constant behavior and remains in this condition while the controlling neuron is maintained as inactive. When the state of the controlling neuron is again loosened, the chaotic activity resumes. Similar results can be obtained by pinning more neurons or by forcing them to a periodic signal.

The control action depends on many factors: the synaptic matrix S_{ij}, the number of synapses per neuron, the number of controlling neurons, and the threshold value. The network controllability is very sensitive to the particular realization of the synaptic matrix S_{ij}. Fixing the threshold, the form of the controlled signal also depends on which neurons are used as control and the time of activation. Most control neurons are very effective, always taking the dynamics to the same attractor, irrespective of the instant in which the control is turned on. Others can lead to different attractors when activated at different time steps. The size of the basin of each attractor and the value of the threshold are the key factors governing this behavior. Specifically, the parameters utilized in our simulations ($N=300$, $n=10$, $T=0$) were chosen because they always give rise to a chaotic network activity that is still susceptible to control, even with a small number of neurons. Adopting these values, control can be achieved with only one neuron in approximately 4% of the runs. Pinning 30 neurons (1/10 of the total network), control is attained in more than 60% of the cases. A more precise account of the effectiveness of the control action and its dependence on the network parameters will be presented in a subsequent contribution.

Some insight on the mechanism of control set forth by the pinning neurons can be gained if we observe how the activity is driven to a period-4 attractor. The Poincaré plot is employed here for this purpose, as seen in Fig. 3. This graph shows the activity at time step $t+1$ plotted against the activity at time step t. The dashed line indicates part of the trajectory without any control. The continuous line shows the trajectory with control. While the uncontrolled path wanders through the plot, the controlled activity is captured in the basin of attraction and, after a few steps, settles as a periodic series with period equal to 4 time steps. Normally this attractor is unstable (around a saddle point), but it can be turned stable by the control action. Figure 3 also shows the approximate directions of the stable and unstable manifolds of the

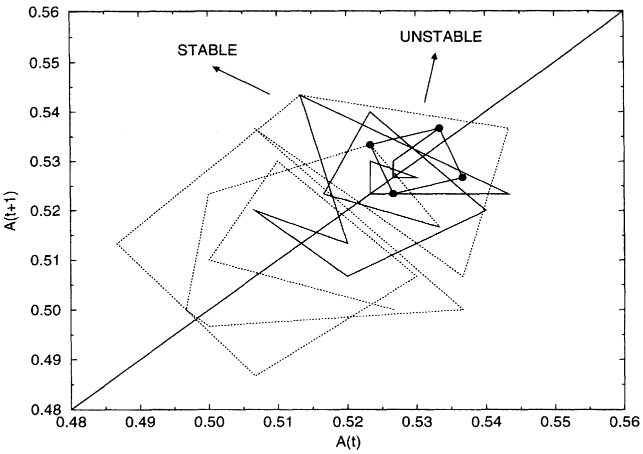

FIG. 3. Poincaré map obtained from an activity time series with threshold $T=0$ and one controlling neuron. The controlled activity is periodic, with period 4. The dashed line corresponds to the uncontrolled trajectory. The continuous line corresponds to the controlled path. General directions of the stable and unstable manifolds are shown.

unstable attractor. The control action pulls the activity to the direction of the stable manifold. Eventually the activity gets trapped by the attractor and remains there while the control is maintained. It is also helpful to examine how the *Hamming distance* to the controlled configuration evolves after the control is turned on. This is shown in Fig. 4 for a typical controlling process. The Hamming distance is defined as $dH=(N-N_c)/N$, where N_c measures, at time step t, the number of neurons with states equal to their corresponding states in the totally controlled configuration. dH is zero when control has effectively taken place. From Fig. 4, it is seen that the process of control is not steadily progressive. When the control is turned on, the average value of dH drops rapidly to a lower value but does not go continuously to zero. It oscillates some time around this lower value and then, quite suddenly, goes to zero and stays there while the control is on.

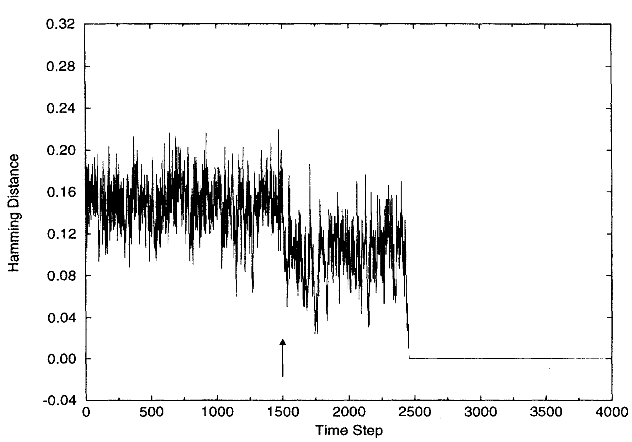

FIG. 4. Hamming distance from the controlled configuration. The arrow shows the time step when the control, with one neuron kept active, is turned on.

Comparing this with the path in the Poincaré plot we can state that the control first spreads its influence through the network forcing the average activity to remain within the basin of attraction of the unstable attractor. This confinement eventually leads the network to a configuration that belongs to the attractor and the systems gets trapped thereafter.

A chaotic trajectory embodies a great number of unstable periodic orbits. The idea of controlling chaos consists in finding a suitable small perturbation that succeeds in stabilizing one of these orbits. The most used method of chaos control, due to Ott, Grebogi, and Yorke [1], proceeds in the following way. An unstable attractor is located together with its local stable and unstable contravariant eigenvectors. These directions in phase space are called the stable and unstable manifolds, respectively. Finding these eigenvectors is relatively easy when the dynamics follows a simple map, with only very few degrees of freedom. For complex systems, such as a neural network, this task can be a matter of discernment. The structure of the attractors is governed by the matrix of synaptic weights. In the simulations reported here, these weights were randomly chosen. However, they can be the result of a prescribed action, such as the Hebbian mechanism of learning [11]. In this process, a number of configurations is presented to the network and the synaptic weights are modified by increasing the strength of those that couple two active neurons. This learning process is believed to be in permanent action in the brain. By this process, the attractors are embedded in the dynamics of the system. They are the "memories" to be recalled by some stimulus received later. The Hebb rule can be applied to store any number of stationary or dynamic objects such as fixed points or cycles [12]. Our model uses a mechanism for memory retrieval that differs from the usual in neural network modeling. Here we have dynamical attractors that can be visited and abandoned by turning on and off the control action. Chaos control is achieved through a match of threshold and stimulus. The threshold is lowered, increasing the mean activity, but the dynamics is kept in the chaotic regime. Then the control is applied. If a match is reached after a transient and the stimulus is maintained, the activity remains periodic or constant. For memory retrieval this model has the advantage of avoiding local minima which is a common problem in spin-glass-like paradigms. The control process can be implemented efficiently when the desired attractor, that is, the memory, is previously known and embedded in the dynamics.

One can conjecture whether some mechanism such as the one described above can be present in biological neural networks. It is known that information is transmitted in the brain as sequences of pulses, the action potentials. These sequences, or spike trains, carry signals from sensory receptors to the brain where they are interpreted and processed in very high rates, with a high efficiency [13]. It is unlikely that the brain uses some complicated algorithm to recall its memories or to make associations. On the other hand, it certainly uses some process that differs from pure trial and error. Real neurons tend to operate near the threshold to be sensitive to input modifications [14]. Lowering the threshold and keeping it low for some time would allow the controlled retrieval of selected trains of spikes, by firing a few specialized neurons. The choice of these neurons can be part of a learning process, together with modifications of synaptic potentials. An intriguing possibility comes related to a hypothesis presented by Crick and Mitchison [15]. They propose that dreams are necessary to remove undesirable modes of excitation in the cortex and reinforce those that are important for survival. It is known that during dreams, in the phase known as paradoxial or REM sleep, the brain activity, measured by EEG series, has a $1/f$ type of spectrum [5]. This might be an indication that, during REM sleep, the brain activity wanders through all allowed neuronal frequencies. It can be argued that this is the stage when the system learns which neuron and which threshold values are to be chosen to fire some desired train of spikes. Recently, we have shown [9] that an analog version of our neural network model can also display intermittent activity, with correlated noise. A subsequent work will try to clarify these issues by the implementation of a learning process while the neural network is in an intermittent regime and with controlled retrieval of memories when it is chaotic.

This research was partially supported by CAPES, CNPq, and FINEP, Brazilian research funding agencies.

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